Mathematical Methods for Engineering

1 Course Overview and Structure

This course operates on a 16-week schedule, consisting of two lectures per week at two hours per lecture (4 hours total weekly). The curriculum is bifurcated into two primary components: a foundational lecture series and independent, directed reading projects. The lecture series establishes the core mathematical framework, while the reading projects allow students to specialize in advanced applications. For detailed information and topic selections regarding the reading projects, please refer to Section 5.

2 Lecture Syllabus

We will follow the primary text by Kreyszig, E. *Advanced Engineering Mathematics* (10th edition).

2.1 Ordinary Differential Equations

First-Order ODEs: Basic concepts, separable, exact, and linear models; existence and uniqueness of solutions.
Second-Order Linear ODEs: Homogeneous equations, constant coefficients, Euler-Cauchy equations, solution by variation of parameters, and modeling of free and forced oscillations.
Higher-Order Linear ODEs and Systems of ODEs: Homogeneous and nonhomogeneous higher-order ODEs; systems of ODEs utilizing matrix methods, phase plane analysis, and qualitative methods for nonlinear systems.
Series Solutions of ODEs and Special Functions: Power series method, Legendre's equation and polynomials, Frobenius method, and Bessel's equation and functions.

2.2 Laplace Transforms

Fundamental concepts including linearity and the first shifting theorem; transforms of derivatives and integrals; unit step function and the second shifting theorem; Dirac's delta function; convolution; differentiation and integration of transforms; applications to systems of ODEs.

2.3 Vector Calculus

Vector Differential Calculus: Inner and cross products, vector and scalar functions, curves, gradient of a scalar field, directional derivatives, divergence, and curl.
Vector Integral Calculus: Line integrals, path independence, Green's Theorem in the plane, surface integrals, the Divergence Theorem of Gauss, and Stokes's Theorem.

2.4 Fourier Analysis and Partial Differential Equations

Fourier Analysis: Fourier series including even and odd functions, and half-range expansions; Sturm-Liouville problems and orthogonal functions; Fourier integrals and transforms.
Partial Differential Equations: Basic concepts; modeling and solution of the Wave Equation

(d'Alembert's solution) and Heat Equation; Dirichlet problem; Laplacian operators in polar, cylindrical, and spherical coordinates; solution of PDEs utilizing Laplace transforms.

3 Evaluation Scheme

The final grade is computed as a weighted sum of the written examinations and the reading project submissions and oral defenses.

Evaluation Component	**Weight (%)**
Reading Project (Viva 1)	25
Partial Exam	25
Reading Project (Viva 2)	25
Final Exam	25

Note that 10% of the reading project points depend on whether the student got the topic of the project approved by August 28, 2026. Failure to do so will result in the loss of these points.

3.1 Evaluation Schedule

All evaluations are conducted during standard class hours.

- **Reading Project (Viva 1):** Week 7 (Lecture 2) and Week 8 (Lecture 1).
- **Partial Exam:** Week 8 (Lecture 2).
- **Reading Project (Viva 2):** Week 15 (Lecture 2) and Week 16 (Lecture 1).
- **Final Exam:** Week 16 (Lecture 2).

4 Weekly Lecture Schedule

Week 1: First-Order ODEs.
Week 2: Second-Order ODEs.
Week 3: *Lecture 1:* Second-Order ODEs. *Lecture 2:* Higher-Order ODEs and Systems.
Week 4: Higher-Order ODEs and Systems.
Week 5: Series Solutions of ODEs and Special Functions.
Week 6: *Lecture 1:* Series Solutions of ODEs and Special Functions. *Lecture 2:* Laplace Transforms.
Week 7: *Lecture 1:* Laplace Transforms. *Lecture 2:* **Viva 1 (Part I)**.
Week 8: *Lecture 1:* **Viva 1 (Part II)**. *Lecture 2:* **Partial Exam**.
Week 9: *Lecture 1:* Laplace Transforms. *Lecture 2:* Vector Differential Calculus.
Week 10: *Lecture 1:* Vector Differential Calculus. *Lecture 2:* Vector Integral Calculus.
Week 11: Vector Integral Calculus.
Week 12: Fourier Analysis.
Week 13: Fourier Analysis.
Week 14: Partial Differential Equations.
Week 15: *Lecture 1:* Partial Differential Equations. *Lecture 2:* **Viva 2 (Part I)**.
Week 16: *Lecture 1:* **Viva 2 (Part II)**. *Lecture 2:* **Final Exam**.

5 Reading Projects

Students will select one of the following tracks for independent reading. Students are free to propose a topic too. Students must create a comprehensive LaTeX document documenting their mathematical derivations and understanding of the topic. During the viva sessions, students will be questioned on their submitted document, and must justify their steps and methodology to test their understanding.

5.1 Reference Materials for Projects

The reading projects draw from the following primary texts:

1. Kreyszig, E. *Advanced Engineering Mathematics.*
2. Oppenheim, A. V., & Willsky, A. S. *Signals and Systems.*
3. Bender, C. M., & Orszag, S. A. *Advanced Mathematical Methods for Scientists and Engineers I: Asymptotic Methods and Perturbation Theory.*
4. Strang, G. *Introduction to Applied Mathematics.*
5. Hughes, T. J. R. *The Finite Element Method.*
6. Powers, J. M., & Sen, M. *Mathematical Methods in Engineering.*
7. Boyd, J. P. *Chebyshev and Fourier Spectral Methods.*
8. Montgomery, D. C. *Design and Analysis of Experiments.*
9. Géron, A. *Hands-On Machine Learning with Scikit-Learn, Keras, and TensorFlow.*

5.2 List of Reading Projects

5.2.1 Basic Probability & Statistical Inference

Source text: Kreyszig.

- **The Reading:** Probability, permutations, combinations, discrete and continuous random variables (Binomial, Poisson, and Normal distributions), point estimation, confidence intervals, hypothesis testing, quality control, and the χ^2 goodness-of-fit test.
- **Real-World Applications:** Baseline data validation, experimental uncertainty assessment.
- **Target Profile:** General Engineering, Biological and Environmental Sciences.

5.2.2 Design of Experiments (DOE)

Source text: Montgomery.

- **The Reading:** Analysis of Variance (ANOVA), full factorial designs, the 2^k factorial design, algebraic decomposition of variance fields, main and interaction effects, orthogonal sign matrices, contrast vectors, and two-level fractional factorial structures.
- **Real-World Applications:** Process optimization, biological growth testing, material property maximization.
- **Target Profile:** Chemical, Industrial, Process, and Environmental Engineering.

5.2.3 Mathematical Foundations of Machine Learning

Source text: Géron.

- **The Reading:** Gradient descent variants (stochastic, mini-batch), multi-layer perceptrons, backpropagation calculus, vanishing and exploding gradients, regularization techniques, and the mathematics of convolutional neural networks (CNNs).
- **Real-World Applications:** Ubiquitous across all disciplines (e.g., predictive diagnostics, computer vision, automated control, pattern recognition).
- **Target Profile:** Computer Science, Data Science, and universally applicable across all engineering profiles.

5.2.4 Continuous Optimization & Linear Programming

Source text: Strang, Kreyszig.

- **The Reading:** Unconstrained optimization, the Simplex Method, and linear constraints.
- **Real-World Applications:** Trajectory optimization, resource allocation.
- **Target Profile:** Industrial and Aerospace Engineering.

5.2.5 Combinatorial Optimization & Graphs

Source text: Strang, Kreyszig.

- **The Reading:** Graph theory, spanning trees, shortest paths, the marriage problem, matching algorithms, maximum network flow, and the transportation problem.
- **Real-World Applications:** Path planning, network routing.
- **Target Profile:** Computer Science and Telecommunications.

5.2.6 Continuous Signals & LTI Systems

Source text: Oppenheim & Willsky.

- **The Reading:** Communication theory, transfer functions, and analog filter design.
- **Real-World Applications:** Wireless communications, acoustics.
- **Target Profile:** Electrical and Mechatronics Engineering.

5.2.7 Discrete Systems & The Z-Transform

Source text: Oppenheim & Willsky, Strang.

- **The Reading:** Difference equations, discrete-time transfer functions, and stability regions.
- **Real-World Applications:** Digital control systems, algorithmic trading.
- **Target Profile:** Computer Engineering and Control Theory.

5.2.8 Dynamical Systems & Stability

Source text: Powers & Sen, Kreyszig.

- **The Reading:** Phase plane analysis, critical points, and qualitative methods for nonlinear systems.
- **Real-World Applications:** Autopilot systems, satellite attitude control.
- **Target Profile:** Control Systems and Aerospace Engineering.

5.2.9 The Finite Element Method (FEM)

Source text: Hughes, Strang.

- **The Reading:** Variational Calculus, the Weak Form, and stiffness matrices.
- **Real-World Applications:** Structural stress analysis, thermal modeling.
- **Target Profile:** Mechanical and Civil Engineering.

5.2.10 Discrete Fourier Analysis & Spectral Methods

Source text: Boyd, Strang.

- **The Reading:** The DFT, the FFT algorithm, and global basis functions.
- **Real-World Applications:** Image compression, quantum mechanics modeling.
- **Target Profile:** Computational Fluid Dynamics and Software Engineering.

5.2.11 Regular Perturbation Theory & Boundary Layer Analysis

Source text: Bender & Orszag.

- **The Reading:** Algebraic and initial-value expansions under a small parameter ϵ, classification of regular vs. singular problems, local coordinate scaling, inner/outer limits, and intermediate asymptotic matching.
- **Real-World Applications:** Fluid boundary layers, localized heat dissipation zones.
- **Target Profile:** Mechanical, Aerospace, and Ocean Engineering.

5.2.12 Singular Perturbation Theory & Multiple-Scale Analysis

Source text: Bender & Orszag.

- **The Reading:** Non-uniform convergence of regular expansions, secular terms, resonance generation, long-term tracking stability, and the formal calculus of splitting fast and slow scales.
- **Real-World Applications:** Non-linear mechanics, satellite orbital decay.
- **Target Profile:** Aerospace Control Systems and Physics.